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Approximate and Conditional Teleportation of an Unknown Atomic State with Dissipative Two-Photon Interaction in Cavity QED System

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Abstract A scheme for approximate and conditional teleportation of an unknown atomic state via dissipative two-photon interaction in cavity QED is proposed. In our scheme, the dissipative two-photon interaction Jaynes-Cummings model is used to realize the approximate and conditional teleportation. We investigate analytically the influence of the cavity mode decay on the teleportation fidelity and show that the high fidelity teleportation can be implemented in dissipative case. Our scheme does not involve the Bell-state measurement and an additional atom, only requiring two atoms and one single-mode cavity. The scheme may be generalized to not only the teleportation of the state of a cavity mode to another mode by means of a single atom but also the teleportation of the state of a trapped ion.

Keywords Teleportation · Cavity QED · Dissipative two-photon interaction

Quantum entanglement plays an key role in quantum information processing, which is one of the most striking features of quantum mechanics. When two subsystems are entangled, the whole state vector cannot be separated into a product of the states of the subsystems and thus the two subsystems are no longer independent even if they are far spatially separated. A measurement on one subsystem not only gives information about the other subsystem, but also provides possibilities of manipulating it. In 1993, Bennett et al. [1] demonstrated that the quantum entanglement can be utilized to teleport an unknown quantum state. At the beginning of the teleportation process, two spin-$1/2$ particles are prepared in the maximally entangled state, that is, Bell state. Then, a joint measurement is performed on the particle to be teleported and one of the entangled pair. In the end, the information of the joint measurement is sent to the other observer through a classical channel and thus he can reproduce the initial state of the teleported particle on the second particle of the entangled pair. Quantum teleportation has been experimentally demonstrated using optical systems [2–4], NMR [5], and trapped ions systems [6–8].

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On the other hand, the cavity QED system is another qualified candidate for demonstrating quantum information processing. Cirac and Parkins [9] have proposed a scheme for the realization of quantum teleportation of an unknown atomic state by using two additional atomic levels of the entangled pair in cavity QED. Davidovich et al. [10] have presented another proposal for the teleportation of an unknown atomic state between two cavities initially prepared in an entangled photon-number states. Other unconditional schemes, none of which have been experimentally implemented, have also been proposed for atomic state teleportation in cavity QED [11–13]. Fang and Cao [14–17] have also presented some schemes for the teleportation of multi-particle state with the Bell-measurement. Recently, Zheng [18–20] has proposed a novel scheme for the approximate conditional teleportation of an unknown atomic state in cavity QED, the distinct feature of which is that it does not require the Bell-state measurement on two qubits. Thus an additional atom is unnecessary and the scheme involve only two atoms interacting with a single-mode cavity field. However, only a time point of system evolution and the corresponding Fidelity implementing the teleportation are given in the scheme. In this paper we propose another novel scheme, which is based on the dissipative two-photon interaction Jaynes-Cummings model in the cavity QED, and then calculate multi-time points and the corresponding Fidelities under consideration of the influence of the cavity mode decay on them and then use them to realize the approximate and conditional teleportation of an unknown atomic state. Naturally, our scheme does also not involve the Bell-state measurement and an additional atom, only requiring two atoms and one single-mode cavity. The Fidelity of the scheme is higher than that of [18–20]. Atom b, which receives the teleported state, is first entangled with the cavity mode. Then, atom a, whose state is to be teleported, interacts with the cavity mode. A measurement on atom a may directly and approximately collapse atom b to the initial state of atom a with the cavity mode left in the vaccum state. The scheme may be generalized to not only the teleportation of the state of a cavity mode to another mode by means of a single atom but also the teleportation of the state of a trapped ion.

In the interaction picture, the atom-cavity resonant interaction is described by the Jaynes-Cummings Hamiltonian of dissipative two-photon interaction

$$H_i = \lambda(a^{+2}S^- e^{\kappa t} + a^2 S^+ e^{-\kappa t}), \tag{1}$$

where a^+ and a are the creation and annihilation operators for cavity field, S^+ and S^- are the raising and lowering operators for atom in the cavity, λ is the atom-field coupling constant and κ denotes the cavity mode decay rate. We assume that atom a to be teleported is initially prepared in the state

$$|\psi\rangle_a = c_g|g\rangle_a + c_e|e\rangle_a, \tag{2}$$

where $|g\rangle_a$ and $|e\rangle_a$ are the ground and excited states of the atom, with c_g and c_e being unknown coefficients. Atom b, to receive the teleported state, is initially prepared in the state $|e\rangle_b$. We send atom b through an initially empty resonant cavity. After interaction time t, the atom exits the cavity and thus the state of the system becomes into

$$|\phi\rangle = \cos\left(\frac{2\sqrt{2}\lambda}{\kappa}\sinh\frac{\kappa t_0}{2}\right)|e\rangle_b|0\rangle - ie^{-\kappa t_0/2}\sin\left(\frac{2\sqrt{2}\lambda}{\kappa}\sinh\frac{\kappa t_0}{2}\right)|g\rangle_b|2\rangle. \tag{3}$$

The atomic velocity is carefully chosen so that $\frac{2\sqrt{2}\lambda}{\kappa}\sinh\frac{\kappa t_0}{2} = \pi/4$ is fulfilled. Then we can obtain the entangled state between the atomic internal states and the cavity modes

$$|\phi\rangle = \frac{1}{\sqrt{2}}[|e\rangle_b|0\rangle - ie^{-\kappa t_0/2}|g\rangle_b|2\rangle]. \tag{4}$$

Now the whole system is in the state

$$|\psi\rangle = \frac{1}{\sqrt{2}}[c_g|g\rangle_a + c_e|e\rangle_a][|e\rangle_b|0\rangle - ie^{-\kappa t_0/2}|g\rangle_b|2\rangle]. \tag{5}$$

Then let atom a interact with the cavity mode. After an interaction time t_1 the system evolves into

$$\begin{aligned}
|\psi'\rangle = \frac{1}{\sqrt{2}}\Bigg\{& c_g|g\rangle_a|e\rangle_b|0\rangle - ic_g e^{-\kappa t_0/2}|g\rangle_b \\
&\times\left[\cos\left(\frac{2\sqrt{2}\lambda}{\kappa}\sinh\frac{\kappa t_1}{2}\right)|g\rangle_a|2\rangle - ie^{-\kappa t_1/2}\sin\left(\frac{2\sqrt{2}\lambda}{\kappa}\sinh\frac{\kappa t_1}{2}\right)|e\rangle_a|0\rangle\right] \\
&+ c_e|e\rangle_b\left[\cos\left(\frac{2\sqrt{2}\lambda}{\kappa}\sinh\frac{\kappa t_1}{2}\right)|e\rangle_a|0\rangle\right. \\
&\left. - ie^{-\kappa t_1/2}\sin\left(\frac{2\sqrt{2}\lambda}{\kappa}\sinh\frac{\kappa t_1}{2}\right)|g\rangle_a|2\rangle\right] \\
&- ic_e e^{-\kappa t_0/2}|g\rangle_b\left[\cos\left(\frac{4\sqrt{3}\lambda}{\kappa}\sinh\frac{\kappa t_1}{2}\right)|e\rangle_a|2\rangle\right. \\
&\left.\left. - ie^{-\kappa t_1/2}\sin\left(\frac{4\sqrt{3}\lambda}{\kappa}\sinh\frac{\kappa t_1}{2}\right)|g\rangle_a|4\rangle\right]\right\}.
\end{aligned} \tag{6}$$

The detection of atom a in the state $|e\rangle_a$ collapses the system consisting of atom b and the cavity field to

$$\begin{aligned}
|\psi''\rangle = N\Bigg[& -c_g e^{\kappa(t_1-t_0)/2}\sin\left(\frac{2\sqrt{2}\lambda}{\kappa}\sinh\frac{\kappa t_1}{2}\right)|g\rangle_b|0\rangle + c_e\cos\left(\frac{2\sqrt{2}\lambda}{\kappa}\sinh\frac{\kappa t_1}{2}\right) \\
&\times|e\rangle_b|0\rangle - ic_e e^{-\kappa t_0/2}\cos\left(\frac{4\sqrt{3}\lambda}{\kappa}\sinh\frac{\kappa t_1}{2}\right)|g\rangle_b|4\rangle\Bigg],
\end{aligned} \tag{7}$$

where N is a normalization factor. When we choose the atomic velocity carefully so that $\frac{2\sqrt{2}\lambda}{\kappa}\sinh\frac{\kappa t_1}{2} = 2k\pi - \pi/4$, with k being integer, and we have $|\cos(\frac{4\sqrt{3}\lambda}{\kappa}\sinh\frac{\kappa t_1}{2})| \le 0.06$. Then the atom b is approximately in the initial state of the atom a,

$$|\psi\rangle_b \approx c_g e^{\kappa(t_1-t_0)/2}|g\rangle_b + c_e|e\rangle_b, \tag{8}$$

with the cavity mode left in the vaccum state. The Fidelity is

$$F = \frac{1}{2}\times\frac{1}{\cos^2(\frac{2\sqrt{2}\lambda}{\kappa}\sinh\frac{\kappa t_1}{2}) + 2|c_e|^2 e^{-\kappa(t_1-t_0)/2}\cos^2(\frac{4\sqrt{3}\lambda}{\kappa}\sinh\frac{\kappa t_1}{2})} \ge 0.995. \tag{9}$$

Through complex computation, we find that when the evolution time is as follows

$$\frac{2\sqrt{2}\lambda}{\kappa}\sinh\frac{\kappa t_1}{2} = \left(20n + \frac{71}{4}\right)\pi, \quad n = 0, 1, \ldots, \tag{10}$$

$\cos(\frac{4\sqrt{3}\lambda}{\kappa}\sinh\frac{\kappa t_1}{2}) = \pm 0.03962 \approx 0$, then the Fidelity $F \ge 0.952$, being lower than that of [18–20], whose value is 0.987. The influence of the cavity mode relative rate $\frac{\kappa}{\lambda}$ on the

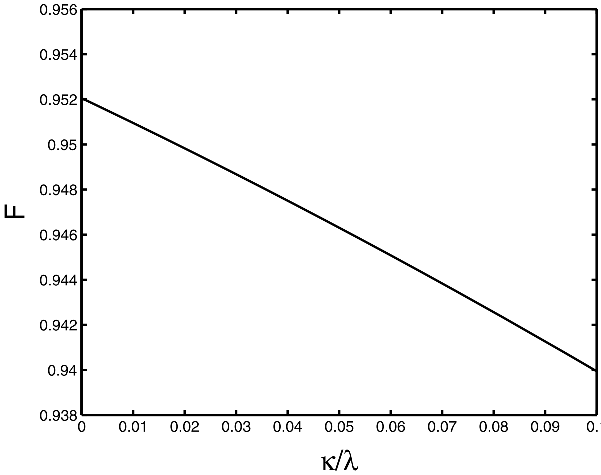

Fig. 1 The relation between the Fidelity F and relative dissipation κ/λ is shown. F decreases with κ/λ increasing

teleportation fidelity F has been shown as Fig. 1, where F decreases with κ/λ increasing. It is clear that we can still calculate many distinct time-evolution points, at which the approximate and conditional teleportation can be implemented. The probability of success is 0.25. The distinct advantage of the scheme is not only that the Bell-state measurement is not required but also that many distinct time-evolution points realizing the conditional teleportation can be given. Moreover, the influence of the mode decay rate on the fidelity has been considered. A direct measurement on atom a may straightly and approximately project atom b on the initial state of atom a with the cavity mode left in the vacuum state. Naturally, The scheme is much simpler than the previous schemes, which require a third atom to preform the Bell-state measurement. At the meantime, the scheme has much richer context than that in [18–20].

It is interesting that the idea can also be used to teleport the state of a cavity mode to another cavity mode using a single atom. We assume that the atom is initially prepared in the state $|e\rangle$. We first send this atom through an initially empty resonant cavity b, and then choose the atomic velocity carefully so that $\frac{2\sqrt{2}\lambda}{\kappa}\sinh\frac{\kappa t_2}{2} = \pi/4$ is fulfilled. Then we have

$$|\varphi\rangle = \frac{1}{\sqrt{2}}[|e\rangle|0\rangle_b - ie^{-\kappa t_2/2}|g\rangle|2\rangle_b]. \tag{11}$$

Assume that the cavity (cavity a) to be teleported is initially prepared in the state

$$|\phi\rangle_a = c_0|0\rangle_a + c_2|2\rangle_a, \tag{12}$$

where c_0 and c_1 are unknown coefficients. Now the whole system is in the state

$$|\psi\rangle = \frac{1}{\sqrt{2}}[c_0|0\rangle_a + c_2|2\rangle_a][|e\rangle|0\rangle_b - ie^{-\kappa t_2/2}|g\rangle|2\rangle_b]. \tag{13}$$

Then let the cavity a interact with the atom for t_3. The system evolves to

$$|\psi'\rangle = \frac{1}{\sqrt{2}}\left\{-ic_0e^{-\kappa t_2/2}|0\rangle_a|g\rangle|2\rangle_b - ic_2e^{-\kappa t_2/2}|2\rangle_b\right.$$

$$\times\left[\cos\left(\frac{2\sqrt{2}\lambda}{\kappa}\sinh\frac{\kappa t_3}{2}\right)|g\rangle|2\rangle_a - i\sin\left(\frac{2\sqrt{2}\lambda}{\kappa}\sinh\frac{\kappa t_3}{2}\right)|e\rangle|0\rangle_a\right]$$

$$+ c_0|0\rangle_b\left[\cos\left(\frac{2\sqrt{2}\lambda}{\kappa}\sinh\frac{\kappa t_3}{2}\right)|e\rangle|0\rangle_a\right.$$

$$\left. - ie^{-\kappa t_2/2}\sin\left(\frac{2\sqrt{2}\lambda}{\kappa}\sinh\frac{\kappa t_3}{2}\right)|g\rangle|2\rangle_a\right]$$

$$+ c_2|0\rangle_b\left[\cos\left(\frac{4\sqrt{3}\lambda}{\kappa}\sinh\frac{\kappa t_3}{2}\right)|e\rangle|2\rangle_a\right.$$

$$\left.\left. - ie^{-\kappa t_2/2}\sin\left(\frac{4\sqrt{3}\lambda}{\kappa}\sinh\frac{\kappa t_3}{2}\right)|g\rangle|4\rangle_a\right]\right\}, \tag{14}$$

where t_3 is the interaction time between the atom and the cavity mode a. The detection of the atom in the state $|e\rangle$ project the system combined the cavity b and the cavity a to

$$|\Psi\rangle = N\left[c_0\cos\left(\frac{2\sqrt{2}\lambda}{\kappa}\sinh\frac{\kappa t_3}{2}\right)|0\rangle_a|0\rangle_b\right.$$

$$- c_2 e^{-\kappa(t_3-t_2)/2}\sin\left(\frac{2\sqrt{2}\lambda}{\kappa}\sinh\frac{\kappa t_3}{2}\right) \times |0\rangle_a|2\rangle_b$$

$$\left. + c_1\cos\left(\frac{4\sqrt{3}\lambda}{\kappa}\sinh\frac{\kappa t_3}{2}\right)|4\rangle_a|0\rangle_b\right], \tag{15}$$

where N is a normalization factor. When $\frac{2\sqrt{2}\lambda}{\kappa}\sinh\frac{\kappa t_3}{2} = (20n + \frac{71}{4})\pi, n = 0, 1, \ldots,$ is fulfilled, $\cos(\frac{4\sqrt{3}\lambda}{\kappa}\sinh\frac{\kappa t_3}{2}) = \pm 0.03962 \approx 0$, cavity b is approximately in the state of the cavity a

$$|\phi\rangle_b \approx c_0 e^{\kappa(t_1-t_0)/2}|0\rangle_b + c_2|2\rangle_b, \tag{16}$$

with cavity a left in the vacuum state. In this case the Fidelity $F' \geq 0.995$. Therefore, the approximate teleportation of a cavity mode to another cavity mode can easily be realized in the dissipative two-photon interaction in QED. Naturally, in our case only a single atom is required.

Now we turn to discuss the experimental feasibility of the proposed scheme. For the Rydberg atoms with principal quantum numbers 50 and 51, the radiative time is $T_r = 3.5 \times 10^{-2}$ s, and the coupling constant is $\lambda = 2\pi \times 25$ kHz [21]. Thus the interaction times of the atoms b and a with the cavity field are $\pi/(4\sqrt{2}\lambda) = 0.35 \times 10^{-5}$ s, $71\pi/(4\sqrt{2}\lambda) = 71 \times 0.35 \times 10^{-5}$ s, respectively, when $\kappa \to 0$. Then we assume that the traveling times of these atoms are 0.35×10^{-5} s and $71 \times 0.35 \times 10^{-5}$ respectively. Thus the time required to complete the whole procedure is about 2.52×10^{-4} s, much shorter than T_r. The decay time of the cavity is of the order $T_c \simeq 2.0 \times 10^{-3}$ s, longer than the required time. Therefore, based on the QED techniques presently or soon, the proposed scheme might be realized.

In fact, the idea can also be generalized to the teleportation of an unknown state of an ion to another ion in a linear trap. Two two-level ions confined in such a linear trap are initially prepared in the vibrational ground state by laser. Ion a, to be teleported, is initially in the electronic state

$$|\psi\rangle_a = c_g|g\rangle_a + c_e|e\rangle_a. \tag{17}$$

Ion b is initially prepared in the electronic excited state $|e\rangle_b$. The second ion is driven by a laser tuned to the second vibrational sideband. In the Lamb-Dicke limit, if the phase of the laser field is adjusted to zero, the dissipative Hamiltonian is [22–26]

$$H = \eta\Omega(a^{+2}S^{-}e^{\kappa t} + a^2 S^{+} e^{-\kappa t}), \tag{18}$$

where a^+ and a are the creation and annihilation operators of the collective motion of the trapped ions, and Ω is the Rabi frequency and κ is the mean decay rate of the ionic collective vibrational modes. The Hamiltonian has the same form of (1) with $\lambda = \eta\Omega$. After an interaction time is satisfied with $\frac{2\sqrt{2}\eta\Omega}{\kappa}\sinh\frac{\kappa t_0}{2} = \pi/4$, ion b and the collective motion is prepared in the entangled state. Then ion a is excited by a laser, also tuned to the second lower vibrational sideband. After an interaction times are determined by $\frac{2\sqrt{2}\eta\Omega}{\kappa}\sinh\frac{\kappa t_1}{2} = (20n + \frac{71}{4})\pi$ $(n = 0, 1, \ldots)$, the detection of ion a in the $|e\rangle_a$ approximately projects the ion b into the initial electronic state of ion a.

In conclusion, we have proposed a scheme for approximately and conditionally teleporting an unknown atomic state in dissipative cavity QED. Our scheme is based on the dissipative two-photon interaction Jaynes-Cummings model. We have given the general formulae of time-points of the system evolution and the corresponding Fidelities, and then used them to realize the approximate and conditional teleportation. In our scheme, we considered the influence of the cavity mode decay rate on the fidelity of the teleportation. Naturally, our scheme does not involve the Bell-state measurement and an additional atom, only requiring two atoms and one single-mode cavity. The scheme has been generalized to not only the teleportation of the state of a cavity mode to another mode by means of a single atom but also the teleportation of the state of a trapped ion. Based on the QED techniques presently or soon, the proposed scheme might be implemented. It is worth noting that our scheme can be generalized to the situation of multi-photon interaction between atoms and the cavity fields, which will be given elsewhere.

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